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Black hole dynamics from instanton strings

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Abstract: A D-5-brane bound state with a self-dual field strength on a 4-torus is considered. In a particular case this model reproduces the D5-D1 brane bound state usually used in the string theory description of 5-dimensional black holes. In the limit where the brane dynamics decouples from the bulk the Higgs and Coulomb branches of the theory on the brane decouple. Contrasting with the usual instanton moduli space approximation to the problem the Higgs branch describes fundamental excitations of the gauge field on the brane. Upon reduction to two dimensions it is associated with the so-called instanton strings. Using the Dirac-Born-Infeld action for the D-5-brane we determine the coupling of these strings to a minimally coupled scalar in the black hole background. The supergravity calculation of the cross section is found to agree with the D-brane absorption probability rate calculation. We consider the near horizon geometry of our black hole and elaborate on the corresponding duality with the Higgs branch of the gauge theory in the large N limit. A heuristic argument for the scaling of the effective string tension is given.

Keywords: D-branes, Black Holes in String Theory, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Over the past two years several issues in black hole physics have been successfully addressed within the framework of string theory (see [1, 2] for reviews and complete lists of references). The black hole dynamics may be recovered from an effective string description [3, 4, 5]. In the dilute gas approximation [6], i.e. when the left- and right-moving modes on the effective string are free and when anti-branes are suppressed, the Bekenstein-Hawking entropy is correctly reproduced. Further, assuming that this effective string couples to the bulk fields with a Dirac-Born-Infeld type action it has been possible to find agreement with the classical cross section calculations for minimally coupled and fixed scalars [5]–[18]. These calculations provide a highly non-trivial test of the effective string model. However, the derivation of the effective string action including its coupling to the bulk fields requires several assumptions. In other words, we would like to deduce this action from first principles as it is the case for similar calculations involving the D-3-brane [19, 20, 21]. One of the purposes of this paper is to fill in this gap.

We shall consider the D-5-brane bound state with a constant self-dual worldvolume field strength on a compact T^4 studied in [23]. This configuration includes as a special case the D5-D1 brane bound state used in the original derivation of the Bekenstein-Hawking entropy formula [3]. The gauge theory fluctuating spectrum associated with this bound state was found to agree with the spectrum derived from open strings ending on the D-5-brane bound state [22, 23, 24]. For this reason the modes associated with the worldvolume fields should be regarded as fundamental excitations of the D-brane system. This includes some modes of the gauge field that are self-dual on T^4 and may be called instantons but should not be interpreted as solitons. We shall see that in the limit where the brane dynamics decouples from the bulk we end up with these instantonic modes plus the modes associated with the movement of the brane system in the transverse directions. They define the Higgs and Coulomb branches of the theory that are shown to decouple in the above limit. The Higgs branch is the one associated with the dynamics of the black hole. We derive from first principles the action for the bosonic fields in the Higgs branch which we call instanton strings action rather then effective string action. We also consider the coupling of these instanton strings to a minimally coupled scalar in the black hole background, finding agreement with the scattering cross section calculation in the supergravity side. This agreement follows because both string and classical calculations have an overlapping domain of validity (this will be our analogue of the double scaling limit introduced by Klebanov [19]), giving a rationale for why both descriptions yield the same result. A deeper explanation is uncovered by Maldacena's duality proposal [25] and subsequent works [26, 27]. We shall elaborate on this proposal. In particular we argue that *the Higgs branch of the large N limit of six-dimensional super Yang-Mills theory with a 't Hooft twist on a compact T^4 is dual to supergravity on $AdS_3 \times S^3 \times T^4$*. Based on this interpretation of the duality conjecture the effective string action should be associated with this large N limit of the theory. We shall give a heuristic derivation of the effective string tension which agrees with previous results [15, 16, 28].

The paper is organised as follows: In section 2 we shall revise the model studied in [23] and analyse the brane dynamics when it decouples from the bulk. The region of validity of both D-brane and supergravity approximations is explained. In section 3 we shall find a minimally coupled scalar in our black hole background and derive the corresponding coupling to the instanton strings. Section 4 is devoted to the supergravity calculation of the scattering cross section as well as the corresponding D-brane absorption probability rate. In section 5 we shall describe the double scaling limit where both calculations are expected to agree. After analysing the near horizon geometry associated with our black hole we consider Maldacena's duality proposal. We give our conclusions in section 6.

2. The model

In this section we shall review the D-brane model associated with our five-dimensional black hole. The dynamics of our D-brane system will be derived by starting from the super Yang-Mills (SYM) action. We shall comment on the validity of such approximation. We review the fluctuating spectrum, study the decoupling of the Higgs and Coulomb branches of the theory when the brane dynamics decouples from the bulk and derive the action for the instanton strings determining the black hole dynamics. We then write the supergravity solution describing the geometry of our black hole and comment on the validity of the supergravity approximation.

Because we are claiming that our model also describes the D5-D1 brane bound state we shall keep refering to this special case as we proceed.

2.1. D-brane phase

We consider a bound state of two D-5-branes wrapped on $S^1 \times T^4$ with coordinates $x^1, \ldots, x^5$ (the generalisation to the case of n D-5-branes is straightforward). Each D-5-brane has winding numbers N_i along S^1, p_i along the x^2-direction and $\bar{p}_i$ along the x^4-direction. Thus, the worldvolume fields take values on the $U(N_1 p_1 \bar{p}_1 + N_2 p_2 \bar{p}_2)$ Lie algebra [29]. In order to have a non-trivial D-5-brane configuration we turn on the worldvolume gauge field such that the corresponding field strength is diagonal and self-dual on T^4. The non-vanishing components are taken to be (we assume without loss of generality that $\tan\theta_1 > \tan\theta_2$)

$$G^0_{23} = G^0_{45} = \frac{1}{2\pi\alpha'}\mathrm{diag}\left(\underbrace{\tan\theta_1, \ldots, \tan\theta_1}_{N_1 p_1 \bar{p}_1 \text{ times}}, \underbrace{\tan\theta_2, \ldots, \tan\theta_2}_{N_2 p_2 \bar{p}_2 \text{ times}}\right), \tag{2.1}$$

where

$$\frac{1}{2\pi\alpha'}\tan\theta_i = \frac{2\pi}{L_2 L_3}\frac{q_i}{p_i} = \frac{2\pi}{L_4 L_5}\frac{\bar{q}_i}{\bar{p}_i}, \tag{2.2}$$

with q_i and $\bar{q}_i$ integers and $L_{\hat{\alpha}} = 2\pi R_{\hat{\alpha}}$ the length of each T^4 circle ($\hat{\alpha} = 2, \ldots, 5$). This vacuum expectation value for the gauge field breaks the gauge invariance to $U(N_1 p_1 \bar{p}_1) \otimes U(N_2 p_2 \bar{p}_2)$. Because the branes are wrapped along the x^1-, x^2- and x^4-directions the gauge invariance is further broken to $U(1)^{N_1 p_1 \bar{p}_1 + N_2 p_2 \bar{p}_2}$. Each D-5-brane carries $Q_{5_i} = N_i p_i \bar{p}_i$ units of D-5-brane charge. Thus, the total D-5-brane charge is

$$Q_5 = N_1 p_1 \bar{p}_1 + N_2 p_2 \bar{p}_2 \,. \tag{2.3}$$

Each brane carries fluxes in the x^2x^3 and x^4x^5 2-tori. The total fluxes are

$$\begin{aligned}\mathcal{F}_{23} &= \frac{1}{2\pi}\int_{T^2_{(23)}} \mathrm{tr}\, G^0 = \left(N_1 q_1 \bar{p}_1 + N_2 q_2 \bar{p}_2\right), \\ \mathcal{F}_{45} &= \frac{1}{2\pi}\int_{T^2_{(45)}} \mathrm{tr}\, G^0 = \left(N_1 p_1 \bar{q}_1 + N_2 p_2 \bar{q}_2\right).\end{aligned} \tag{2.4}$$

These fluxes induce a 't Hooft twist on the fields [30, 31, 32, 33], i.e. the worldvolume fields obey twisted boundary conditions on T^4. Also, due to this vacuum expectation value for the field strength the D-5-branes carry other D-brane charges. There are $Q_3 = \mathcal{F}_{45}$ D-3-brane charge units associated with D-3-branes parallel to the (123)-directions, and $Q_{3'} = \mathcal{F}_{23}$ D-3-brane charge units associated with D-3-branes parallel to the (145)-directions. Furthermore, the instanton number associated with the background field strength is non-zero. As a consequence the bound state carries the D-string charge [34]

$$Q_1 = N_{ins} = \frac{1}{16\pi^2}\int_{T^4} \mathrm{tr}\left(G^0 \wedge G^0\right) = N_1 q_1 \bar{q}_1 + N_2 q_2 \bar{q}_2 \,. \tag{2.5}$$

It is now clear how we can obtain a bound state with the same charges as de D5-D1 brane system. We just have to set the fluxes in (2.4) to zero and the charges Q_5 and Q_1 are given by (2.3) and (2.5), respectively. For example, if we set $q_1 = \bar{q}_1 = 1$ and $q_2 = \bar{q}_2 = -1$, then $N_1 p_1 = N_2 p_2$, $N_1 \bar{p}_1 = N_2 \bar{p}_2$ and the D-string charge is $Q_1 = N_1 + N_2$.

Now we consider the region of validity of the D-brane description of our bound state. Troughout this paper we shall always assume that $g \ll 1$ so closed string effects beyond tree level are suppressed. Also, we assume that the size of T^4 is small, i.e. $L_{\hat{\alpha}} \sim \sqrt{\alpha'}$. The effective coupling constant for D-brane string perturbation theory is usually gN for N D-branes on top of each other. However the presence of a condensate on the D-brane worldvolume induces a factor $\sqrt{1 + (2\pi\alpha' G^0)^2}$ in the effective coupling [35]. Thus, in our case the effective string coupling reads

$$g_{eff} = g N_i p_i \bar{p}_i \sqrt{1 + (2\pi\alpha' G^0_{23})^2}\,\sqrt{1 + (2\pi\alpha' G^0_{45})^2} \equiv \frac{r_i^2}{\alpha'} \,. \tag{2.6}$$

The length scales r_i will enter the supergravity solution below and we assume for simplicity $r_1 \sim r_2$. D-brane perturbation theory is valid for [6]

$$r_i \ll 1 \,, \tag{2.7}$$

where the r_i are now written in string units. In this region open string loop corrections may be neglected and the dynamics for the low lying modes on the brane is determined by the Dirac-Born-Infeld (DBI) action. Our tool to study this region of parameters is the 10-dimensional SYM action reduced to six dimensions. The corresponding bosonic action is

$$S_{YM} = -\frac{1}{g^2_{YM}}\int d^6x\, \mathrm{tr}\left\{\frac{1}{4}(G_{\alpha\beta})^2 + \frac{1}{2}(\partial_\alpha \phi_m + i[B_\alpha, \phi_m])^2 - \frac{1}{4}[\phi_m, \phi_n]^2\right\}, \tag{2.8}$$

where $\alpha, \beta = 0, \ldots, 5$ and $m, n = 6, \ldots, 9$. We are taking the fields to be hermitian matrices and the field strength is given by $G_{\alpha\beta} = \partial_\alpha B_\beta - \partial_\beta B_\alpha + i[B_\alpha, B_\beta]$. The Yang-Mills coupling constant is related to the D-5-brane tension T_5 by

$$g_{YM}^2 = \frac{1}{(2\pi\alpha')^2 T_5} = (2\pi)^3 g\alpha' . \tag{2.9}$$

Note that in our conventions both B_α and ϕ_m have dimension length^{-1}. This action is the leading term in the α' expansion of the DBI action. In this approximation we have

$$\left(2\pi\alpha' G^0_{\hat{\alpha}\hat{\beta}}\right)^2 \ll 1 \quad \Rightarrow \quad |\tan\theta_i| \ll 1 \, . \tag{2.10}$$

Physically this condition may be obtained from the requirement

$$M_{5_i} \gg M_{3_i}, \;\; M_{3'_i}, \;\; M_{1_i} \, , \tag{2.11}$$

where M_{3_i} and $M_{3'_i}$ are the masses of the D-3-branes dissolved in the D-5-brane with mass M_{5_i} and similarly for M_{1_i}. If this condition does not hold we expect the D-5-branes to be bent or deformed [36]. Because we are assuming that $L_{\hat{\alpha}} \sim \sqrt{\alpha'}$ we see from eqn. (2.2) that the condition (2.10) gives $p_i \gg |q_i|$ and $\bar{p}_i \gg |\bar{q}_i|$. We remark that in this limit there is perfect agreement between the string and SYM spectrum derived in [23].

Next we review the fluctuating spectra of our SYM theory. The starting point is to expand the action around the background (2.1). The result is

$$\begin{aligned} S_{SYM} = -\frac{1}{4g_{YM}^2} \int d^6x \; \mathrm{tr} \Big\{ & -2A^\alpha D^2 A_\alpha - 4iA^{\hat{\beta}}[G^0_{\hat{\beta}\hat{\alpha}}, A^{\hat{\alpha}}] - 2\phi^m D^2 \phi_m + \\ & + 2i(D_\alpha A_\beta - D_\beta A_\alpha)[A^\beta, A^\alpha] + 4i\phi^m D_\alpha[\phi_m, A^\alpha] - \\ & - [A_\alpha, A_\beta]^2 - 2[A_\alpha, \phi_m]^2 - [\phi_m, \phi_n]^2 \Big\} \, , \end{aligned} \tag{2.12}$$

where we have done the following splitting of the gauge field:

$$\begin{aligned} B_\alpha &= B^0_\alpha + A_\alpha \, , \\ G_{\alpha\beta} &= G^0_{\alpha\beta} + F_{\alpha\beta} \, , \\ G^0_{\alpha\beta} &= \partial_\alpha B^0_\beta - \partial_\beta B^0_\alpha + i[B^0_\alpha, B^0_\beta] \, , \\ F_{\alpha\beta} &= D_\alpha A_\beta - D_\beta A_\alpha + i[A_\alpha, A_\beta] \, , \end{aligned} \tag{2.13}$$

with $D_\alpha \equiv \partial_\alpha + i[B^0_\alpha, \;\;]$. The quantum fields A_α and ϕ_m are in the adjoint representation of $U(N_1 p_1 \bar{p}_1 + N_2 p_2 \bar{p}_2)$ and A_α satisfies the background gauge fixing condition $D_\alpha A^\alpha = 0$. These fields obey twisted boundary conditions on $S^1 \times T^4$ [30, 31, 32, 33]. We have $(\beta \neq 0)$

$$A_\alpha(x^\beta + L_\beta) = \Omega_\beta A_\alpha(x^\beta) \Omega_\beta^{-1}, \tag{2.14}$$

and similarly for ϕ_m. The Ω's are called multiple transition functions and take values on $U(N_1p_1\bar{p}_1+N_2p_2\bar{p}_2)$. They are given by

$$\Omega_\alpha = \mathrm{Diag}(\Omega_\alpha^{(1)},\Omega_\alpha^{(2)})\,, \tag{2.15}$$

where $\Omega_\alpha^{(i)}\in\ U(N_ip_i\bar{p}_i)$ and in terms of $U(p_i)\otimes U(\bar{p}_i)\otimes U(N_i)$ matrices reads

$$\begin{aligned}
\Omega_1^{(i)} &= \mathbf{1}_{p_i}\otimes\mathbf{1}_{\bar{p}_i}\otimes V_{N_i}\,,\\
\Omega_2^{(i)} &= \exp\left[-\pi i n^i_{2\hat{\beta}}x^{\hat{\beta}}/L_{\hat{\beta}}\right]\mathbf{V}_{p_i}\otimes\mathbf{1}_{\bar{p}_i}\otimes\mathbf{1}_{N_i}\,,\\
\Omega_3^{(i)} &= \exp\left[-\pi i n^i_{3\hat{\beta}}x^{\hat{\beta}}/L_{\hat{\beta}}\right](\mathbf{U}_{p_i})^{q_i}\otimes\mathbf{1}_{\bar{p}_i}\otimes\mathbf{1}_{N_i}\,,\\
\Omega_4^{(i)} &= \exp\left[-\pi i n^i_{4\hat{\beta}}x^{\hat{\beta}}/L_{\hat{\beta}}\right]\mathbf{1}_{p_i}\otimes\mathbf{V}_{\bar{p}_i}\otimes\mathbf{1}_{N_i}\,,\\
\Omega_5^{(i)} &= \exp\left[-\pi i n^i_{5\hat{\beta}}x^{\hat{\beta}}/L_{\hat{\beta}}\right]\mathbf{1}_{p_i}\otimes(\mathbf{U}_{\bar{p}_i})^{\bar{q}_i}\otimes\mathbf{1}_{N_i}\,.
\end{aligned} \tag{2.16}$$

The matrices $\mathbf{V}_{N_i}$ are the $N_i\times N_i$ shift matrices and similarly for $\mathbf{V}_{p_i}$ and $\mathbf{V}_{\bar{p}_i}$. The matrices $\mathbf{U}_{p_i}$ are given by

$$\mathbf{U}_{p_i} = \mathrm{diag}\left(1, e^{2\pi i\frac{1}{p_i}},\ldots,e^{2\pi i\frac{p_i-1}{p_i}}\right), \tag{2.17}$$

and similarly for $\mathbf{U}_{\bar{p}_i}$. The tensors $n^i_{\hat{\alpha}\hat{\beta}}$ are called the twist tensors and are given by

$$n^i_{\hat{\alpha}\hat{\beta}} = \begin{pmatrix} 0 & q_i/p_i & 0 & 0\\ -q_i/p_i & 0 & 0 & 0\\ 0 & 0 & 0 & \bar{q}_i/\bar{p}_i\\ 0 & 0 & -\bar{q}_i/\bar{p}_i & 0\end{pmatrix}. \tag{2.18}$$

In order to analyse the spectrum it is convenient to decompose the fields A_α and ϕ_m as

$$A_\alpha = \begin{pmatrix} a^1_\alpha & b_\alpha\\ b^\dagger_\alpha & a^2_\alpha\end{pmatrix},\qquad \phi_m = \begin{pmatrix} c^1_m & d_m\\ d^\dagger_m & c^2_m\end{pmatrix}. \tag{2.19}$$

The fields a^i_α and c^i_m are in the adjoint representation of $U(N_ip_i\bar{p}_i)$ and the fields b_α and d_m in the fundamental representation of $U(N_1p_1\bar{p}_1)\otimes U(\overline{N_2p_2\bar{p}_2})$. Substituting the ansatz (2.19) in the action (2.12) and keeping only the quadratic terms in the fields together with the boundary conditions (2.14) we may derive the spectrum of the theory. The result is resumed in table 1. Note that we are using the complex coordinates $z_k = (x^{2k}-ix^{2k+1})/\sqrt{2}$ with $k=1,2$ to express the fields $b_{\hat{\alpha}}$. It is important to realize that the functions $\chi^r_{m_1m_2}$ determining the mode expansion of the fields b_α and d_m on T^4 have the same 'status' as the usual modes $e^{ik_{\hat{\alpha}}x^{\hat{\alpha}}}$. They form a basis for functions satisfying the twisted boundary conditions obeyed by these fields on T^4 and are expressed in terms of Θ-functions. Also the quadratic operator $\hat{M}$ is the analogue of $(\partial_{\hat{\alpha}})^2$. Each eigenvalue of this operator has a degeneracy $n_L\bar{n}_L$ associated with the number of Landau levels in the system.

Fields	Quadr. operators	Modes	On-shell cond.	No. of d.o.f.
$a^i_{\hat{\alpha}}$	$-(\partial_\sigma)^2-(\partial_{\hat{\alpha}})^2$	$e^{ik_\sigma x^\sigma+ik_{\hat{\alpha}}x^{\hat{\alpha}}}$	$k_\sigma^2+k_{\hat{\alpha}}^2=0$	$4N_ip_i\bar{p}_i$
b_{z_k}	$-(\partial_\sigma)^2+(\hat{M}-4\pi f)$	$e^{ik_\sigma x^\sigma}\chi^r_{m_1m_2}(x^{\hat{\alpha}})$	$k_\sigma^2+\lambda^-_{m_1m_2}=0$	$4n_L\bar{n}_L$
$b_{\bar{z}_k}$	$-(\partial_\sigma)^2+(\hat{M}+4\pi f)$	$e^{ik_\sigma x^\sigma}\chi^r_{m_1m_2}(x^{\hat{\alpha}})$	$k_\sigma^2+\lambda^+_{m_1m_2}=0$	$4n_L\bar{n}_L$
c^i_m	$-(\partial_\sigma)^2-(\partial_{\hat{\alpha}})^2$	$e^{ik_\sigma x^\sigma+ik_{\hat{\alpha}}x^{\hat{\alpha}}}$	$k_\sigma^2+k_{\hat{\alpha}}^2=0$	$4N_ip_i\bar{p}_i$
d_m	$-(\partial_\sigma)^2+\hat{M}$	$e^{ik_\sigma x^\sigma}\chi^r_{m_1m_2}(x^{\hat{\alpha}})$	$k_\sigma^2+\lambda_{m_1m_2}=0$	$8n_L\bar{n}_L$

Table 1: Spectrum of the theory presented in a form suitable for reduction to two dimensions. We have imposed the Coulomb gauge condition $A_0=0$ and used the fact that $D_\alpha A^\alpha = 0$ to fix A_1 (for the mode of b_{z_k} with $\lambda^-=0$ this gives $A_1=0$). The operator $\hat{M}$ is given by $\hat{M}=(i\partial_{\hat{\alpha}}+\pi J_{\hat{\alpha}\hat{\beta}}x^{\hat{\beta}})^2$ with $J_{\hat{\alpha}\hat{\beta}}=(n^1_{\hat{\alpha}\hat{\beta}}-n^2_{\hat{\alpha}\hat{\beta}})/(L_{\hat{\alpha}}L_{\hat{\beta}})$. The functions $\chi^r_{m_1m_2}$ are eigenfunctions of the operator $\hat{M}$ with eigenvalues $\lambda_{m_1m_2}=4\pi f(m_1+m_2+1)$ where $4\pi f=(\tan\theta_1-\tan\theta_2)/(\pi\alpha')$ and $\lambda^\pm_{m_1m_2}=\lambda_{m_1m_2}\pm 4\pi f$. The index r in the functions $\chi^r_{m_1m_2}$ runs from 1 to $n_L\bar{n}_L$ with $n_L=|p_1q_2-p_2q_1|$ and $\bar{n}_L=|\bar{p}_1\bar{q}_2-\bar{p}_2\bar{q}_1|$. The index σ runs from 0 to 1.

In table 1 we wrote the mode expansion for the various fields but some care is necessary because each field carries Lie algebra indices. Consider first the case of the fields $b_{\hat{\alpha}}$ and d_m. The corresponding modes in the table are defined on a $S^1_{eff}\times T^4_{eff}$ 5-torus while a given $a\bar{b}$ Lie algebra component of these fields takes values on $S^1\times T^4$ determined by a given segment of $S^1_{eff}\times T^4_{eff}$. The different Lie algebra components are then related by the boundary conditions. In the case of these fields we have S^1_{eff} with a length $L_{eff}=N_1N_2L_1$ and T^4_{eff} with radi $(p_1p_2R_2,R_3,\bar{p}_1\bar{p}_2R_4,R_5)$. A similar comment applies to the $a^i_{\hat{\alpha}}$ and c^i_m fields but now S^1_{eff} has radius N_iR_1 and T^4_{eff} has radi $(p_iR_2,R_3,\bar{p}_iR_4,R_5)$. To be more explicite we consider the modes on T^4 of the fields b_{z_k} with the lowest eigenvalue $\lambda^-=0$ (i.e. $m_1=m_2=0$). These are the only modes coming from the fields $b_{\hat{\alpha}}$ and d_m that are associated with massless particles in two dimensions. We write a given Lie algebra component of the fields, say the $1\bar{1}$ component, as ($b^{1\bar{1}}_{\bar{z}_k}=0$)

$$
\begin{aligned}
b^{1\bar{1}}_{z_1} &= \frac{1}{2\pi\alpha'}\sum_{r=1}^{n_L\bar{n}_L}\xi^r_1(x^\sigma)\chi^r(x^{\hat{\alpha}})\,,\\
b^{1\bar{1}}_{z_2} &= \frac{1}{2\pi\alpha'}\sum_{r=1}^{n_L\bar{n}_L}\xi^r_2(x^\sigma)\chi^r(x^{\hat{\alpha}})\,.
\end{aligned}
\tag{2.20}
$$

The complex fields ξ^r_k are defined on an effective circle with radius $R_{eff}=N_1N_2R_1$ and $\chi^r(x^{\hat{\alpha}})$ takes values on the T^4_{eff} defined above. The fields $b^{1\bar{1}}_{z_k}$ take values on $S^1\times T^4$ and all the other Lie algebra components may be obtained from this one by using the boundary conditions (2.14).[1] It is important to realize that the fields $b^{a\bar{b}}_{z_k}$ are operators in the quantum theory and therefore the fields ξ^r_k are also quantum operators.

[1]We are assuming here that p_1,p_2 and $\bar{p}_1,\bar{p}_2$ are co-prime. It is not difficult to drop this condition [23].

2.1.1. Theory on the brane in the decoupling limit

We consider the limit where the brane dynamics decouples from the bulk. This limit corresponds to taking $\alpha' \to 0$ and $N_i \to \infty$, i.e. we are considering the large N theory in the infrared limit. Noting that the radi of T^4 scale as $\sqrt{\alpha'}$ we conclude that the massive Kałuża-Klein modes on T^4 associated with the fields $a^i_{\hat{\alpha}}$ and c^i_m decouple from the theory in the above limit. Also, with the exception of the massless modes associated with the fields b_{z_k} in (2.20) all the excitations associated with $b_{\hat{\alpha}}$ and d_m decouple. Thus, we are left with the massless excitations associated with the fields $c^i_m(x^\sigma)$, $a^i_{\hat{\alpha}}(x^\sigma)$ and $\xi^r_k(x^\sigma)$.

To analyse the resulting theory it is convenient to consider the T-dual six-dimensional theory with worldvolume given by the string directions x^σ and by the tranverse space to the D-5-brane system $\mathbb{E}^4$. We follow very closely the analysis given in [36]. In fact, our model provides an explicite realization of the results there derived. We start with $N = 2$ SUSY in $D = 6$ but the self-dual background field strength on the T^4 breaks half of the supersymmetries leaving $N = 1$ SUSY in $D = 6$. There are two possible multiplets, the vector multiplet and the hypermultiplet [37]. The fields c^i_m correspond to the gauge independent degrees of freedom of gauge bosons and therefore fall into a vector multiplet. The fields $a^i_{\hat{\alpha}}$ are 4 scalars and fall into a hypermultiplet. The resulting theory is just two copies of $10D$ SYM compactified on T^4, each copy with gauge group $U(N_i p_i \bar{p}_i)$. So far we have the field content of $N = 2$ SUSY in $D = 6$. The fields c^i_m in the vector multiplets are left invariant under the $SO(4)_I$ rotational symmetry associated with the T^4 while the fields $a^i_{\hat{\alpha}}$ in the hypermultiplets transform as $\bar{\mathbf{4}}$ under this symmetry. The theory is not $SO(4)_I \simeq SU(2)_L \otimes SU(2)_R$ invariant because the background field strength breaks this symmetry. If this field was not (anti) self-dual we would be left with a $U(1) \otimes U(1)$ symmetry corresponding to rotations in the x^2, x^3 and x^4, x^5 directions. For (anti) self-dual fields this symmetry gets enhanced to $U(2) \simeq SU(2) \otimes U(1)$ [31]. The action of this group in the $z_k, \bar{z}_k$ coordinates is generated by

$$\mathrm{i}\sigma_\mathbf{3} \otimes \mathbf{1}\,, \qquad \mathrm{i}\sigma_\mathbf{3} \otimes \sigma_\mathbf{3}\,, \qquad \mathrm{i}\mathbf{1} \otimes \sigma_\mathbf{2}\,, \qquad \mathrm{i}\sigma_\mathbf{3} \otimes \sigma_\mathbf{1}\,, \tag{2.21}$$

where the first generator corresponds to the $U(1)$ factor. Thus, the resulting $N = 1$ theory as a $U(2)$ R-symmetry. We still have to consider the complex fields ξ^r_k. For each r, they describe 4 scalar fields and therefore fall into a hypermultiplet [38]. The fields $\xi^r = (\xi^r_1\ \xi^r_2)$ transform as $\bar{\mathbf{2}}$ under the $U(2)$ R-symmetry.

The reduction of the theory to two dimensions results in a theory with $N = 4$ SUSY in $2D$. Now both the hypers and vectors have 4 scalar fields. They are distinguished by the different transformation properties under R-symmetries. The theory has an extra $SO(4)_E \simeq SU(2)_{\tilde{L}} \otimes SU(2)_{\tilde{R}}$ R-symmetry that leaves the scalars in the hypermultiplets unchanged but acts on the scalars in the vector multiplets. This theory has two supersymmetric branches, the Higgs branch where the hypers are excited and the Coulomb branch where the vectors are excited. Supersymmetry implies that there is no couplings between the vector and hyper multiplets [36]. We shall describe the different branches of the theory in the next subsection, for now let us just note that in the Higgs phase the fields $a^i_{\hat\alpha}$ condense and the only independent degrees of freedom are associated with the fields ξ^r_k.

All this resembles the moduli space approximation to the dynamics of the D5-D1 brane bound state. The Higgs branch describing the moduli of instantons on T^4 and the Coulomb branch the fluctuations of the system in the transverse space. However, there is a crucial difference in our description. The modes of the quantum fields b_{z_k} that survived the decoupling limit are self-dual on T^4 and in that sense deserve to be called instantons but rather then being interpreted as solitons they should be interpreted as fundamental modes of the fields (just as the standard $e^{ik_{\hat\alpha}x^{\hat\alpha}}$ modes). In other words, we are *not* quantising the collective coordinates of a soliton (instanton). There are two reasons for this: Firstly, they are the field theory realization of the low lying modes corresponding to open strings with ends on the D-5-branes with a different background field strength. Thus, they are as fundamental as the other modes corresponding to the a^i_α and c^i_m fields associated with open strings ending on the same D-5-branes. Secondly, these instantons do not really have a size in the sense that there is no moduli associated with its size. In fact, all the dependence of b_{z_k} on T^4 is through $x^{\hat\alpha}/L_{\hat\alpha}$. Thus, if the volume of T^4 is scaled the fields scale uniformly [39]. Also, this means that we can take the limit $L_{\hat\alpha} \to 0$ and the field configuration remains well defined.

There is a potential problem when we take the size of T^4 to be of order one in string units. Because the fields b_{z_k} are $x^{\hat\alpha}$ dependent we could expect that string derivative corrections to the DBI (or SYM) action become important [40, 41]. It turns out that for $\Delta\theta \equiv \theta_1 - \theta_2 \ll 1$ which holds when the DBI corrections are suppressed the derivative corrections are also suppressed. To see this recall that the wave functions $\chi_{m_1 m_2}$ in table 1 may be generated by the creation operators $a^\dagger_k$ with [23]

$$\begin{aligned} a^\dagger_k &= \frac{1}{i\sqrt{2\pi f}}\left(\partial_{z_k} - \pi f \bar z_k\right), \qquad k = 1\,,2\,, \\ a_l &= \frac{1}{i\sqrt{2\pi f}}\left(\partial_{\bar z_l} + \pi f z_l\right), \qquad l = 1\,,2\,, \end{aligned} \tag{2.22}$$

and $[a_l, a^\dagger_k] = \delta_{lk}$. We then have $\langle z, \bar z | m_1 m_2\rangle = \chi_{m_1 m_2}(z, \bar z)$, where $|m_1 m_2\rangle$ is normalised to unit. Considering for example a typical derivative correction term like $\sqrt{\alpha'}\partial_2\chi_0$ we obtain from (2.22)

$$\sqrt{\alpha'}\partial_2\chi_0 = \mathrm{i}\sqrt{\frac{\Delta\theta}{4\pi}}\left(\chi_{1,0} + \sqrt{\frac{\Delta\theta}{4\pi}}\frac{\mathrm{x}^3}{\sqrt{\alpha'}}\chi_0\right), \tag{2.23}$$

which is negligible for $\Delta\theta \ll 1$ (note that $4\pi f = \Delta\theta/(\pi\alpha')$). Thus, our field theory description is valid for $V_4 \sim \alpha'^2$. As an aside note that the disagreement between the string and gauge theory normalisation for the masses of the excitations on the brane bound state may be due to this derivative corrections. A fact that as been suggested in [33].

To summarise, rather then doing a moduli space approximation we have a vacuum state defined by the background field strength $G^0_{\hat{\alpha}\hat{\beta}}$ which is a (constant) instanton. The quantum fluctuations around this vacuum are well defined by open strings ending on the D-5-brane bound state and their low energy field theory realization is resumed in table 1. We have a quantum mechanical description of the excitations around the instanton vacuum state pretty much as the description of the D-5-brane/D-string configuration given by Callan and Maldacena [42]. In the decoupling limit $\alpha' \to 0$ we ended up with the quantum fields $c^i_m(x^\sigma)$, $a^i_{\hat{\alpha}}(x^\sigma)$ and $\xi^r_k(x^\sigma)$.

2.1.2. Higgs and Coulomb branches

Now we describe the Higgs and Coulomb branches of the theory. We shall see that in the decoupling limit here considered these branches decouple [43, 44]. We want to define a supersymmetric branch of the theory on the brane such that it will describe the dynamics of the black holes considered in the next section. These black holes will be appropriately identified with some state of the theory on the brane which may or may not preserve some supersymmetry.

Since the fields $b^{a\bar{b}}_{z_k}$ originate a self-dual field strength on T^4 the resulting compactified theory is supersymmetric. However, when we consider the interactions between these fields it is seen that the field strength $F_{\hat{\alpha}\hat{\beta}}$ is no longer self-dual. To next order in the fields $b^{a\bar{b}}_{z_k}$ the self-duality condition holds if the fields $a^i_{\hat{\alpha}}$ condense. They are determined by

$$
\begin{aligned}
a^i_{\hat{\beta}} &= \Box^{-1}\partial_{\hat{\alpha}} S^i_{\hat{\alpha}\hat{\beta}}\,,\\
(S^1_{\hat{\alpha}\hat{\beta}})^{ab} &= -i\left[\left(b^{a\bar{c}}_{\hat{\alpha}} b^{\dagger\bar{c}b}_{\hat{\beta}} - b^{a\bar{c}}_{\hat{\beta}} b^{\dagger\bar{c}b}_{\hat{\alpha}}\right) - \tfrac{1}{2}\epsilon_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\theta}}\left(b^{a\bar{c}}_{\hat{\gamma}} b^{\dagger\bar{c}b}_{\hat{\theta}} - b^{a\bar{c}}_{\hat{\theta}} b^{\dagger\bar{c}b}_{\hat{\gamma}}\right)\right],\\
(S^2_{\hat{\alpha}\hat{\beta}})^{ab} &= -i\left[\left(b^{\dagger\bar{a}c}_{\hat{\alpha}} b^{c\bar{b}}_{\hat{\beta}} - b^{\dagger\bar{a}c}_{\hat{\beta}} b^{c\bar{b}}_{\hat{\alpha}}\right) - \tfrac{1}{2}\epsilon_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\theta}}\left(b^{\dagger\bar{a}c}_{\hat{\gamma}} b^{c\bar{b}}_{\hat{\theta}} - b^{\dagger\bar{a}c}_{\hat{\theta}} b^{c\bar{b}}_{\hat{\gamma}}\right)\right],
\end{aligned}
\tag{2.24}
$$

where $\Box \equiv \partial^2_{\hat{\alpha}}$. We could have $N_i p_i \bar{p}_i$ commuting components of the free fields $a^i_{\hat{\beta}}$ and still have self-duality. The boundary conditions (2.14) imply that these components would have to be on the diagonal or on a shifted diagonal when the fields are expressed in terms of $U(p_i) \otimes U(\bar{p}_i) \otimes U(N_i)$ matrices. This would give 4 massless particles defined on an effective length $N_i L_1$. This contribution is subleading in the large N limit considered here. The condensation of the fields is a familiar fact. It corresponds to requiring the D-term $[A_{\hat{\alpha}}, A_{\hat{\beta}}]^2$ in the action (2.12) to vanish which does not happen when we consider just the fields $b^{a\bar{b}}_{z_k}$ (the cubic term in $A_{\hat{\alpha}}$ vanishes). Further, when the fields ξ^r_k are excited the commutator term $[A_{\hat{\alpha}}, \phi_m]^2$ in the action gives a mass term to the c^i_m fields and vice-versa. Thus, in the low energy limit we have the Higgs branch with the fields ξ^r_k excited and the Coulomb branch with the fields c^i_m excited.

A more careful analysis is as follows: We start by considering a classical field configuration that defines a supersymmetric branch of the theory, i.e. we consider the moduli space of supersymmetric classical vacua. This corresponds to set all the D-terms of the theory to zero. The D-terms are

$$V_1 = -\mathrm{tr}[\phi_m, \phi_n]^2 \,, \qquad V_2 = -\mathrm{tr}[A_{\hat{\alpha}}, \phi_m]^2 \,, \qquad V_3 = -\mathrm{tr}[A_{\hat{\alpha}}, A_{\hat{\beta}}]^2 \,. \tag{2.25}$$

The unusual minus signs are because we took our fields to be hermitian. V_3 vanishes because the $a^i_{\hat{\alpha}}$ fields condense, therefore we are just left with V_1 and V_2. They become

$$\begin{aligned} V_1 &= -\mathrm{tr}[c^1_m, c^1_n]^2 - \mathrm{tr}[c^2_m, c^2_n]^2 \,, \\ V_2 &= -\mathrm{tr}[a^1_{\hat{\alpha}}, c^1_m]^2 - \mathrm{tr}[a^2_{\hat{\alpha}}, c^2_m]^2 + 2\mathrm{tr}[(c^1_m)^2 b_{\hat{\alpha}} b^\dagger_{\hat{\alpha}} + (c^2_m)^2 b^\dagger_{\hat{\alpha}} b_{\hat{\alpha}} - 2c^2_m b^\dagger_{\hat{\alpha}} c^1_m b_{\hat{\alpha}}] \,. \end{aligned} \tag{2.26}$$

Now there are only two possibilities (apart from the trivial case $\phi_m \sim \mathbf{1}$): (1) $c^i_m = 0$ and then ξ^r_k may be generic. This is the Higgs branch. (2) The c^i_m are generic but all commute. Because the branes are wrapped the boundary conditions require that these fields take the form

$$c^i_m \sim (V_{p_i})^r \otimes (V_{\bar{p}_i})^s \otimes (V_{N_i})^t \,, \tag{2.27}$$

where the V's are the shift matrices and r, s, t are integers. The claim is that in order to vanish V_2 we need $\xi^r_k = 0$. This may be seen by noting that if $\xi^r_k \neq 0$ the condensate formed by the fields $a^i_{\hat{\alpha}}$ will give a non-vanishing $\mathrm{tr}[a^i_{\hat{\alpha}}, c^i_m]^2$.

We conclude that classically we either have a Higgs or a Coulomb branch. Quantum mechanically we consider fluctuations of the fields around the classical vacua that obey the D-flatness conditions. Each branch defines a different superconformal field theory. This has to be the case because a $(4,4)$ superconformal field theory has a $SU(2) \otimes SU(2)$ group of left- and right-moving symmetries that must leave the scalars in the theory invariant [45]. In the Higgs branch this group originates from the $SO(4)_E$ symmetry while in the Coulomb branch from the $SO(4)_I$ symmetry (which is broken to $U(2)$ in the Higgs branch).

2.1.3. Instanton strings action

The action for the fields ξ^r_k in the Higgs branch may be obtained by replacing the field configurations corresponding to (2.20) and (2.24) in the action (2.12). We normalise the functions χ^r according to

$$\int_{T^4_{eff}} d^4x \, (\chi^r)^* \, \chi^s = (2\pi\sqrt{\alpha'})^4 \delta^{rs} \,, \qquad r, s = 1, \ldots, n_L \bar{n}_L \tag{2.28}$$

which is well defined in the limit $\alpha' \to 0$ because $R_{\hat{\alpha}} \sim \sqrt{\alpha'}$. Defining $4n_L\bar{n}_L$ real fields ζ^r from the ξ^r_k complex fields and replacing the field configuration corresponding to (2.20) in the action (2.12) we obtain after some algebra the following 1+1-dimensional free action

$$S = -\frac{T_{ins}}{2} \int dt \int_0^{L_{eff}} dx^1 \sum_{r=1}^{4n_L\bar{n}_L} \partial_\sigma \zeta^r \partial^\sigma \zeta^r \,, \qquad (2.29)$$

where

$$T_{ins} = \frac{1}{2\pi\alpha' g} \,, \qquad L_{eff} = 2\pi R_1 N_1 N_2 \,, \qquad f = 4n_L\bar{n}_L \,, \qquad (2.30)$$

are the instanton strings tension, the effective length and the number of bosonic species in our model, respectively. In order to compare these results with the effective string model for the D5-D1 system used in the literature let us write the corresponding quantities

$$T_{eff} = \frac{1}{2\pi\alpha' g\sqrt{Q_5 Q_1}} \,, \qquad L_{eff} = 2\pi R_1 Q_5 Q_1 \,, \qquad f = 4 \,. \qquad (2.31)$$

The particular combination of Q_1 and Q_5 in T_{eff} has been derived in [15, 16, 28]. We shall argue in section 5 that by taking the large N limit of our field theory the instanton strings tension gets normalised reproducing an effective string tension which agrees with this prediction. Using the result $Q_5 Q_1 = N_1 N_2 n_L \bar{n}_L$ which holds for the D5-D1 system we see that our results for L_{eff} and f are not necessarily in contradiction with (2.31). Note that for the D5-D1 brane bound state described by our field configuration we must have $f \geq 16$.[2] The case $f = 16$ corresponds to the example given after equation (2.5) if we set $p_i = \bar{p}_i = 1$. If this is the case one could argue that our results are not reliable because the DBI corrections are important. This is certainly true but things may not be as bad as they look. The reason is that the supersymmetric configuration that we have found depends exclusively on the boundary conditions satisfied by the fields and on the self-duality condition. The former depends only on the gauge invariance of the theory and it is certainly independent of the specific Lagrangian describing the dynamics of the system. The latter is sufficient to show that our field configuration preserves a fraction of the supersymmetries and it is also independent of the specific Lagrangian. In fact, the self-duality condition was shown in [46] to be a sufficient condition to minimise the non-abelian DBI action proposed by Tseytlin [47]. These arguments together with the string analysis given in [23] provide evidence for the validity of the supersymmetric field configuration even when the DBI corrections are expected to be important. Thus, we do not expect L_{eff} and f to be altered. What changes is the interacting theory and not the free action (2.29).

We should now worry about the supersymmetry completion of the action (2.29). In [36] it was shown that this action takes the form

$$S = -\frac{T_{ins}}{2} \int d^2x \left(G_{rs}(\zeta) \partial_\sigma \zeta^r \partial^\sigma \zeta^s + \text{fermions} \right) , \qquad (2.32)$$

[2]We remark that this fact may be related to the fact that our field strength background does not have a minimal integer instanton number. N_{ins} is always a multiple of 2. Generalising our results to arbitrary integer instanton number may allow the possibility $f = 4$.

where G_{rs} is a hyperkähler metric. This defines the superconformal field theory describing the Higgs phase. From our knowledge of the b_{z_k} and $a^i_{\hat{\alpha}}$ field configurations corresponding to (2.20) and (2.24) one could in principle attempt to find the ζ^r corrections to the flat metric $G_{rs} = \delta_{rs}$ in (2.29).

We end this subsection by considering the dilute gas regime. As in (2.11) stability of our D-brane bound state requires that the energy associated with the string modes should be much smaller then all the energy scales associated with the D-brane bound state. This gives the condition (a similar derivation of the dilute gas regime was given in [48])

$$\frac{N_{L,R}}{R_1} \ll M_{1_i} \ \left(\ll M_{3_i},\ M_{3'_i} \ll M_{5_i} \right), \tag{2.33}$$

where $N_{L,R}$ are the left- and right-moving momenta carried by the instanton strings along the x^1-direction (note that, e.g. $N'_R = N_1 N_2 N_R$ is the level of the right-moving sector because $L_{eff} = 2\pi R_1 N_1 N_2$). Condition (2.33) gives

$$r_0,\ r_n \ll r_i \tan\theta_i \,, \tag{2.34}$$

where we define the length scales r_n and r_0 according to [50]

$$r_n^2 = r_0^2 \sinh^2\beta \,, \qquad N_{L,R} = \frac{R_1^2 V_4}{4g^2\alpha'^4} r_0^2 e^{\pm 2\beta}, \tag{2.35}$$

with V_4 the volume of T^4. The condition (2.34) defines the dilute gas regime derived in [6].

2.2. Supergravity phase

The supergravity solution associated with our D-brane bound state is a solution of the type IIB supergravity equations of motion. The corresponding bosonic action is

$$\begin{aligned} S_{IIB} = \frac{1}{2\kappa_{10}^2} \Bigg\{ \int d^{10}x\sqrt{-g} \Bigg[e^{-2\phi_{10}} \left(R + 4(\nabla\phi_{10})^2 - \frac{1}{2.3!}\mathcal{H}^2 \right) - \frac{1}{2}(\partial\chi)^2 - \\ - \frac{1}{2.3!}(\mathcal{F}_3 - \chi\mathcal{H})^2 - \frac{1}{4.5!}\mathcal{F}_5'^2 \Bigg] - \frac{1}{2}\int \mathcal{A}_4 \wedge \mathcal{H} \wedge \mathcal{F}_3 \Bigg\} , \end{aligned} \tag{2.36}$$

where κ_{10} is the ten-dimensional gravitational coupling, $\mathcal{F}'_5 = d\mathcal{A}_4 + \frac{1}{2}(\mathcal{B}\wedge\mathcal{F}_3 - \mathcal{A}_2\wedge\mathcal{H})$ is a self-dual 5-form, $\mathcal{H} = d\mathcal{B}$ and $\mathcal{F}_3 = d\mathcal{A}_2$. The fields χ, $\mathcal{A}_2$ and $\mathcal{A}_4$ are the 0-, 2- and 4-form R-R potentials and the field $\mathcal{B}$ the 2-form NS-NS potential. ϕ_{10} is the dilaton field with its zero mode subtracted. The NS-NS background fields describing our bound state are

$$\begin{aligned} ds^2 &= H^{\frac{1}{2}} \left[H^{-1}\left(-dt^2 + dx_1^2\right) + \tilde{H}^{-1}\left(dx_2^2 + \cdots + dx_5^2\right) + ds^2\left(\mathbb{E}^4\right) \right], \\ e^{2\phi} &= H\tilde{H}^{-2}, \\ \mathcal{B} &= -\frac{\tilde{H}^{-1}}{r^2} \sum_i r_i^2 \sin\theta_i \cos\theta_i \left(dx_2 \wedge dx_3 + dx_4 \wedge dx_5 \right), \end{aligned} \tag{2.37}$$

where

$$
\begin{aligned}
H &= 1 + \frac{r_1^2 + r_2^2}{r^2} + \frac{(r_1 r_2 \sin\Delta\theta)^2}{r^4}\,, \\
\tilde{H} &= 1 + \frac{r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2}{r^2}\,,
\end{aligned}
\tag{2.38}
$$

with r the radial coordinate on $\mathbb{E}^4$. The constants θ_i and r_i are defined in (2.2) and (2.6), respectively. The exact form of the R-R fields is rather complicated because the Chern-Simons terms for this solution do not vanish. We write all non-vanishing components of the R-R fields keeping only the corresponding leading order terms at infinity. The result is

$$
\begin{aligned}
\mathcal{F}_{at1} &\sim d\left(\frac{1}{r^2}\right)_a \sum_i r_i^2 \sin^2\theta_i + \mathcal{O}\left(\frac{1}{r^5}\right), \\
\mathcal{F}_{abc} &\sim -\star\, d\left(\frac{1}{r^2}\right)_{abc} \sum_i r_i^2 \cos^2\theta_i + \mathcal{O}\left(\frac{1}{r^2}\right), \\
\mathcal{F}_{at123} = \mathcal{F}_{at145} &\sim d\left(\frac{1}{r^2}\right)_a \sum_i r_i^2 \sin\theta_i\cos\theta_i + \mathcal{O}\left(\frac{1}{r^5}\right), \\
\mathcal{F}_{abc23} = \mathcal{F}_{abc45} &\sim -\star\, d\left(\frac{1}{r^2}\right)_{abc} \sum_i r_i^2 \sin\theta_i\cos\theta_i + \mathcal{O}\left(\frac{1}{r^2}\right),
\end{aligned}
\tag{2.39}
$$

where $\star$ is the dual operation with respect to the Euclidean metric on $\mathbb{E}^4$. This solution corresponds to the vacuum state of our D-brane bound state.

Next we obtain the D5-D1 brane solution as a special case. All we have to do is to require that the D-3-brane charges vanish, i.e.

$$
r_1^2\cos\theta_1\sin\theta_1 + r_2^2\cos\theta_2\sin\theta_2 = 0\,,
\tag{2.40}
$$

and redefine the parameters r_5 and r_1 as

$$
\begin{aligned}
r_5^2 &\equiv r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2\,, \\
r_1^2 &\equiv r_1^2\sin^2\theta_1 + r_2^2\sin^2\theta_2\,.
\end{aligned}
\tag{2.41}
$$

We have then that $H = H_1 H_5$, $\tilde{H} = H_5$ (note that $r_1 r_2 \sin\Delta\theta \equiv r_5 r_1$) and the resulting solution simplifies dramatically (specially the R-R fields) to the well known D5-D1 solution. Note that by taking $\theta_1 = 0$ and $\theta_2 = \pi/2$ we also obtain the D5-D1 solution. However, our field theory description does not hold because the gauge field diverges. In this case the correct description is given by the D-5-brane/D-string picture [42].

We may add some momentum along the string direction in (2.37). In the D-brane picture this corresponds to excite the left- and right-moving sectors of the instanton strings theory. If we keep in the dilute gas region defined in (2.34) and further assume that

$$
r_0^2,\ r_n^2 \ll r_1 r_2 \sin\Delta\theta\,,
\tag{2.42}
$$

then all the fields in (2.37), (2.39) remain unchanged but the metric which becomes

$$ds^2 = H^{\frac{1}{2}} \left[H^{-1} \left(-dt^2 + dx_1^2 + \left(\frac{r_0}{r}\right)^2 \left(\cosh\beta dt - \sinh\beta dx_1 \right)^2 \right) + \right.$$
$$\left. + \tilde{H}^{-1} \left(dx_2^2 + \cdots + dx_5^2 \right) + \left(1 - \left(\frac{r_0}{r}\right)^2 \right)^{-1} dr^2 + r^2 d\Omega_3^2 \right], \quad (2.43)$$

where r_0, r_n and β are defined in (2.35). Note that in the case of the D5-D1 system the condition (2.42) follows from (2.34) and (2.10). For given values of r_n and r_0 the total left- and right-moving momenta along the strings are completely fixed. This means that the state of the instanton strings is described by the microcanonical ensemble. Using the asymptotic density of states for a conformal field theory with $4n_L\bar{n}_L$ species of bosons and fermions we obtain the usual matching with the Bekenstein-Hawking entropy

$$S_{BH} = \frac{A_3}{4G_N^{(5)}} \left(r_n r_1 r_2 \sin\Delta\theta \right) = 2\pi \left(\sqrt{N_L} + \sqrt{N_R} \right) \sqrt{N_1 N_2 n_L \bar{n}_L}\,. \quad (2.44)$$

This agreement occurs for $N_{L,R}N_1N_2 \gg n_L\bar{n}_L$. This fact may be interpreted in the following way. We may approximate the microcanonical ensemble by the canonical ensemble with the left- and right-moving temperatures [6]

$$T_{L,R} = \frac{1}{\pi R_1} \sqrt{\frac{N_{L,R}}{N_1 N_2 n_L \bar{n}_L}} = \frac{r_0 e^{\pm\beta}}{2\pi r_1 r_2 \sin\Delta\theta}\,. \quad (2.45)$$

The occupation number for a given mode is then easily calculated in the canonical ensemble. This approximation is valid for $T_{L,R} \gg E_g$, where $E_g \sim (R_1N_1N_2)^{-1}$ is the energy gap in the field theory side. Physically this means that in the thermodynamical description the energy spectrum may be regarded as continuous. Replacing for the values of $T_{L,R}$ we obtain precisely the condition $N_{L,R}N_1N_2 \gg n_L\bar{n}_L$. Thus, the non-extreme case (2.43) is associated with a thermal state of the instanton strings.

Now we comment on the region of validity of the supergravity approximation. We keep $g \ll 1$ in order to suppress closed string loop effects. We are also making an α' expansion. Thus, the length scales in our solution have to be much larger then one in string units, i.e.

$$r_0,\ r_n,\ r_i,\ \sqrt{r_1 r_2 \sin\Delta\theta} \gg 1\,. \quad (2.46)$$

The supergravity approximation is valid for processes involving energy scales such that $\omega l_{max} \ll 1$ where l_{max} is the maximal length scale [36]. We conclude that the D-brane and supergravity phases are mutually exclusive. Considering the last condition in (2.46) for the region of validity of the supergravity phase we have in terms of the D-brane system $g\sqrt{N_1N_2n_L\bar{n}_L} \gg 1$. We shall see below that consistency between the supergravity and D-brane phases requires $n_L\bar{n}_L$ not to be very large. Hence we have (for $N_1 \sim N_2$)

$$gN_i \gg 1\,. \quad (2.47)$$

Since g is small we conclude that $N_i \gg 1$. Thus, the supergravity phase is associated with a large N_i D-brane system.

Next we show that to compare with the supergravity phase it is perfectly consistent to neglect the massive string states in the field theory side. This corresponds to the $\alpha' \to 0$ decoupling limit where these states become infinitely massive. The condition to neglect such modes is

$$T_{L,R} \ll \frac{1}{\sqrt{\alpha'}} \quad \Leftrightarrow \quad r_n^2,\ r_0^2 \ll (r_1 r_2 \sin\Delta\theta)^2 \frac{1}{\alpha'}\,. \tag{2.48}$$

Using the conditions (2.42) and (2.46) it is seen that (2.48) holds. Thus, in the supergravity side we do not expect to find effects caused by these fields and it is consistent to drop them in the field theory approach (note that in this case we are not protected by supersymmetry as it was the case in [23]).

Another check of consistency between both descriptions is concerned with the mass gap. In the field theory description this equals $(N_1 N_2 R_1)^{-1}$, while in the supergravity side it is given by the inverse of the temperature such that the specific heat is of order unit [49, 51, 52, 53]. This condition gives

$$\delta M \sim \frac{G_N^{(5)}}{(r_1 r_2 \sin\Delta\theta)^2} \sim (N_1 N_2 n_L \bar{n}_L R_1)^{-1}. \tag{2.49}$$

Thus, we cannot have $n_L \bar{n}_L$ very big. For the D5-D1 brane system this fact brings us to the case where the DBI corrections are important that we have discussed in subsection 2.1.3. In the more general case we may have $n_L \bar{n}_L \sim 1$ while keeping $p_i \gg |q_i|$ and $\bar{p}_i \gg |\bar{q}_i|$ (for example this happens for $q_i = \bar{q}_i = 1$ and $p_1 = p_2 - 1$, $\bar{p}_1 = \bar{p}_2 - 1$ while keeping $p_i \gg 1$ and $\bar{p}_i \gg 1$).

3. Minimally coupled scalar

In this section we shall find a minimally coupled scalar in the supergravity backgrounds of subsection 2.2. We shall follow the same strategy of [9] by reducing the type IIB action to five-dimensions. Then we will linearise the DBI action and generalise the result to the non-abelian case in order to determine the coupling of the minimally coupled scalar to the instanton strings.

3.1. Reduction to five dimensions

To find a minimally coupled scalar in our black hole background we reduce the action (2.36) with the following metric ansatz [9]

$$ds^2 = e^{\frac{4}{3}\phi_5} g_{ab} dx^a dx^b + e^{2\nu_5}\left(dx^1 + \mathcal{A}_a^{(K)} dx^a\right)^2 + e^{2\nu} \delta_{\hat{\alpha}\hat{\beta}} dx^{\hat{\alpha}} dx^{\hat{\beta}}, \tag{3.1}$$

where g_{ab} is the five-dimensional Einstein metric.[3] Truncating the action (2.36) such that the only non-vanishing form fields are those appearing in the solution (2.37), (2.39) and assuming as it is the case that $\mathcal{A}_{a1}$, $\mathcal{A}_{a1\hat{\alpha}\hat{\beta}}$ and $\mathcal{A}^{(K)}_a$ are electric we obtain the following 5-dimensional action (S_1 and S_2 were vanishing in the case considered in [9])

$$
\begin{aligned}
S = & \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R-(\partial\Phi)^2-\frac{4}{3}(\partial\lambda)^2-4(\partial\nu)^2-\frac{1}{4}e^{\frac{8}{3}\lambda}\left(\mathcal{F}^{(K)}_{ab}\right)^2-\right.\\
& \left.-\frac{1}{2.2!}e^{-\frac{4}{3}\lambda+4\nu}\left(\mathcal{F}_{ab1}\right)^2-\frac{1}{2.3!}e^{\frac{4}{3}\lambda+4\nu}\left(\mathcal{F}_{abc}\right)^2\right]+S_1+S_2\,,\\
S_1 = & \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[-\frac{1}{2.2!}e^{-4\nu}\left(\partial_a\mathcal{B}_{\hat{\alpha}\hat{\beta}}\right)^2-\right.\\
& \left.-\frac{1}{4.2!2!}e^{-\frac{4}{3}\lambda}\left({\mathcal{F}'}_{ab1\hat{\alpha}\hat{\beta}}\right)^2-\frac{1}{4.2!3!}e^{\frac{4}{3}\lambda}\left({\mathcal{F}'}_{abc\hat{\alpha}\hat{\beta}}\right)^2\right],\\
S_2 = & \frac{1}{4\kappa_5^2}\int d^5x\frac{1}{2!2!}\epsilon^{\hat{\alpha}\hat{\beta}\hat{\theta}\hat{\gamma}}\epsilon^{abcde}\left(\frac{1}{3!}\mathcal{A}_{a1\hat{\alpha}\hat{\beta}}\mathcal{F}_{bcd}\mathcal{H}_{e\hat{\theta}\hat{\gamma}}-\frac{1}{2!2!}\mathcal{A}_{ab\hat{\alpha}\hat{\beta}}\mathcal{F}_{cd1}\mathcal{H}_{e\hat{\theta}\hat{\gamma}}\right),
\end{aligned}
\tag{3.2}
$$

where κ_5 is the five-dimensional gravitational coupling and

$$
\Phi=\phi_{10}-2\nu=\phi_5+\frac{\nu_5}{2}\,,\qquad \lambda=\nu_5-\frac{\Phi}{2}=\frac{3}{4}\nu_5-\frac{\phi_5}{2}\,.
\tag{3.3}
$$

The 5-form ${\mathcal{F}'}_5$ reduces to

$$
\begin{aligned}
{\mathcal{F}'}_{ab1\hat{\alpha}\hat{\beta}} &= \mathcal{F}_{ab1\hat{\alpha}\hat{\beta}}+\frac{1}{2}\left(\mathcal{B}_{\hat{\alpha}\hat{\beta}}\mathcal{F}_{ab1}-2\mathcal{A}_{1[a}\mathcal{H}_{b]\hat{\alpha}\hat{\beta}}\right),\\
{\mathcal{F}'}_{abc\hat{\alpha}\hat{\beta}} &= \mathcal{F}_{abc\hat{\alpha}\hat{\beta}}+\frac{1}{2}\left(\mathcal{B}_{\hat{\alpha}\hat{\beta}}\mathcal{F}_{abc}-3\mathcal{A}_{[ab}\mathcal{H}_{c]\hat{\alpha}\hat{\beta}}\right),
\end{aligned}
\tag{3.4}
$$

and the ten-dimensional self-duality condition $\star\mathcal{F}'=\mathcal{F}'$ becomes

$$
{\mathcal{F}'}_{abc\hat{\alpha}\hat{\beta}}=\frac{1}{2!2!}\sqrt{-g}e^{-\frac{4}{3}\lambda}\epsilon_{abcde}\epsilon_{\hat{\alpha}\hat{\beta}\hat{\theta}\hat{\gamma}}{\mathcal{F}'}^{de}{}_{1\hat{\theta}\hat{\gamma}}\,.
\tag{3.5}
$$

We conclude that the field Φ (dilaton field in the six-dimensional theory) is minimally coupled.

3.2. Coupling to the instanton strings

The coupling of the scalar field Φ to the instanton strings may be found following a similar approach to the D-3-brane case [19, 20]. Start with the DBI action for the D-5-brane written in the static gauge

$$
\begin{aligned}
S_{DBI} &= -T_5\int d^6x\; e^{-\phi_{10}}\sqrt{-\det\left(\hat{g}+\mathcal{G}\right)}\;+\text{RR couplings}\,,\\
\mathcal{G}_{\alpha\beta} &= 2\pi\alpha' G_{\alpha\beta}-\hat{\mathcal{B}}_{\alpha\beta}\,,\\
\hat{g}_{\alpha\beta} &= g_{\alpha\beta}+2g_{m(\alpha}\partial_{\beta)}X^m+g_{mn}\partial_\alpha X^m\partial_\beta X^n.
\end{aligned}
\tag{3.6}
$$

[3] In this subsection the indices $a,b,\ldots$ run over 0,6,...9, otherwise they are ten-dimensional space-time indices.

As for $\hat{g}_{\alpha\beta}$ the field $\hat{\mathcal{B}}_{\alpha\beta}$ is the pull back to the D-5-brane worldvolume of the NS-NS 2-form potential. We set $\mathcal{B}$ to zero and expand the metric around flat space: $g_{ab} = \eta_{ab} + h_{ab}$. Then we expand the action (3.6) keeping the quadratic terms in the worldvolume fields and the linear terms in the bulk fields. Defining the scalar fields $\phi^m = X^m/(2\pi\alpha')$ the result is

$$\begin{aligned} S_{DBI} &\sim -(2\pi\alpha')^2 T_5 \int d^6x \left[(1-\phi_{10}) \left(\frac{1}{4} G^2_{\alpha\beta} + \frac{1}{2} \partial_\alpha \phi^m \partial^\alpha \phi_m \right) - \right. \\ &\qquad\qquad\qquad \left. - \frac{1}{2} h^{\alpha\beta} T_{\alpha\beta} + \frac{1}{2} h_{mn} \partial_\alpha \phi^m \partial^\alpha \phi^n \right] , \\ T_{\alpha\beta} &= G_\alpha{}^\theta G_{\beta\theta} - \frac{1}{4}\eta_{\alpha\beta} G^2 + \partial_\alpha \phi^m \partial_\beta \phi_m - \frac{1}{2}\eta_{\alpha\beta} \partial_\theta \phi^m \partial^\theta \phi_m , \end{aligned} \tag{3.7}$$

where the indices are raised and lowered with respect to the Minkowski metric and $T_{\alpha\beta}$ is the energy momentum tensor of the abelian YM action (free terms in (3.7)). The coupling between the fields Φ and B_α is determined by the coupling of ϕ_{10} and $h^{\alpha\beta}$ to B_α. Therefore we drop the last term in the action (3.7). The obvious generalisation of the interacting action to the $U(N)$ case is

$$\begin{aligned} S_{int} &= \frac{1}{g^2_{YM}} \int d^6x \left[\phi_{10} \mathrm{tr} \left(\frac{1}{4} G^2_{\alpha\beta} + \frac{1}{2} \left(\partial_\alpha \phi_m + i[B_\alpha, \phi_m] \right)^2 - \frac{1}{4} [\phi_m, \phi_n]^2 \right) + \frac{1}{2} h^{\alpha\beta} T_{\alpha\beta} \right] , \\ T_{\alpha\beta} &= \mathrm{tr} \left(G_\alpha{}^\theta G_{\beta\theta} - \frac{1}{4}\eta_{\alpha\beta} G^2 + (\partial_\alpha \phi^m + i[B_\alpha, \phi^m])(\partial_\beta \phi_m + i[B_\beta, \phi_m]) - \right. \\ &\qquad \left. - \frac{1}{2} \eta_{\alpha\beta} (\partial_\theta \phi_m + i[B_\theta, \phi_m])^2 + \frac{1}{4} \eta_{\alpha\beta} [\phi_m, \phi_n]^2 \right) . \end{aligned} \tag{3.8}$$

The situation is analogous to the calculation involving the D-3-brane [19, 20]. Note that it is straightforward to write the supersymmetric completion of (3.8) because ϕ_{10} couples to the SYM action and $h^{\alpha\beta}$ to the corresponding energy-momentum tensor. We are just writing the interacting terms that follow from the SYM action but there will be DBI type corrections as well as there may be modifications to the energy momentum tensor imposed by conformal invariance [21].

In the linear approximation the scalar fields in the ansatz (3.1) are identified with the tensor $h_{\alpha\beta}$ according to

$$h = h^{\hat\alpha}{}_{\hat\alpha} = 8\nu \qquad h_{00} = -\frac{4}{3}\phi_5 , \qquad h_{11} = 2\nu_5 . \tag{3.9}$$

Keeping only the interacting terms with the field Φ that are quadratic in the worldvolume fluctuating field A_α we have (note that $G_{\alpha\beta} = G^0_{\alpha\beta} + F_{\alpha\beta}$)

$$S_{int} = \frac{1}{g^2_{YM}} \int d^6x \ \Phi \ \mathrm{tr} \left(\frac{1}{2} F_{\sigma\hat\alpha} F^{\sigma\hat\alpha} \right) . \tag{3.10}$$

As explained before we are just considering processes involving the massless excitations on the brane and keeping only the fields associated with the instanton strings. A similar calculation to the one in subsection 2.1.3 gives the following interacting term between Φ and the instanton strings:

$$S_{int} = -\frac{T_{ins}}{2} \int dt \int_0^{L_{eff}} dx^1 \; \Phi \sum_{r=1}^{4n_L \bar{n}_L} \partial_\sigma \zeta^r \partial^\sigma \zeta^r, \tag{3.11}$$

where T_{ins} and L_{eff} are given in (2.30). The factor multiplying the integral is important.

4. Cross section

In this section we calculate the cross section for the scattering of the D-brane bound state by the scalar particle Φ both in the supergravity and D-brane picture. We shall consider the supergravity solutions corresponding to the D-brane system vacuum and thermal states.

4.1. Classical calculation

The equation of motion satisfied by the scalar field in the background (2.37) is for the s-wave mode

$$\left[r^{-3} \frac{d}{dr} r^3 \frac{d}{dr} + \omega^2 \left(1 + \frac{r_1^2 + r_2^2}{r^2} + \frac{(r_1 r_2 \sin \Delta\theta)^2}{r^4} \right) \right] \Phi(r) = 0\,. \tag{4.1}$$

Writing $\Phi = \rho^{-3/2} \Psi(\rho)$ where $\rho = \omega r$ we have

$$\left[\frac{d^2}{d\rho^2} + 1 - \frac{3/4 - \omega^2 \left(r_1^2 + r_2^2\right)}{\rho^2} + \frac{\omega^4 (r_1 r_2 \sin \Delta\theta)^2}{\rho^4} \right] \Psi(\rho) = 0\,. \tag{4.2}$$

For $\rho \gg \omega r_i \sin \Delta\theta$ (i.e. $r \gg r_i \sin \Delta\theta$) we may neglect the $\mathcal{O}(1/\rho^4)$ term in comparison with the $\mathcal{O}(1/\rho^2)$ terms. In the low energy limit $\omega r_i \ll 1$ we are considering, the differential equation satisfied by $\Psi(\rho)$ becomes

$$\left[\frac{d^2}{d\rho^2} + 1 - \frac{3}{\rho^2} \right] \Psi(\rho) = 0\,, \tag{4.3}$$

which is solved in terms of Bessel functions of degree one.[4]

If we perform instead the coordinate transformation $y = \omega r_1 r_2 \sin \Delta\theta / (2r^2)$, the differential equation eq. (4.1) becomes

$$\left[\frac{d^2}{dy^2} + \omega \frac{r_1 r_2 \sin \Delta\theta}{2y} + \omega^2 \frac{r_1^2 + r_2^2}{4y^2} + \omega^3 \frac{r_1 r_2 \sin \Delta\theta}{8y^3} \right] \Phi(y) = 0\,. \tag{4.4}$$

[4] An alternative resolution is to keep the ωr_i terms, solve the differential equation in terms of Bessel functions of degree $\pm\sqrt{1 - \omega^2 \left(r_1^2 + r_2^2\right)}$ and take the limit $\omega r_i \ll 1$ at the end. Within our approximation the final result is the same.

The last term may be neglected for $y \gg \omega \sin\Delta\theta$ (i.e. $r \ll r_i$). In the coordinate $z = \sqrt{2y\omega r_1 r_2 \sin\Delta\theta}$ and with $\Phi = z\Upsilon(z)$ we have in the limit $\omega r_i \ll 1$

$$\left[z^2\frac{d^2}{dz^2} + z\frac{d}{dz} + (z^2-1)\right]\Upsilon(z) = 0\,, \tag{4.5}$$

which is again solved in terms of Bessel functions of degree one. Since $\Delta\theta \ll 1$ we conclude that both the equations (4.3) and (4.5) have a large overlapping domain and the corresponding solutions may be patched together.[5]

The cross section may be calculated by using the flux method [6]. In the near zone we require a purely infalling solution at $r = 0$ and match it to the solution in the far zone. The result is

$$\begin{aligned} &\text{Near region: } \Phi = z\Big(J_1(z) + N_1(z)\Big)\,,\\ &\text{Far region:} \qquad \Phi = -\frac{4}{\pi\rho}J_1(\rho)\,, \end{aligned} \tag{4.6}$$

where J_1 and N_1 are Bessel and Neumann functions, respectively. The cross section is obtained from the ratio between the flux at the horizon and the incoming flux at infinity

$$\sigma_{abs} = \frac{4\pi}{\omega^3}\frac{\mathcal{F}_h}{\mathcal{F}_\infty^{inc}} = \pi^3\omega\left(r_1 r_2 \sin\Delta\theta\right)^2. \tag{4.7}$$

The calculation of the cross section for the non-extreme case is similar to the calculation presented in [6]. In the far region the solution is the same as in the previous case and in the near region Φ is expressed in terms of hypergeometric functions. The result is

$$\sigma = A_h \frac{\pi\omega}{2}\frac{e^{\frac{\omega}{T_H}} - 1}{\left(e^{\frac{\omega}{2T_L}} - 1\right)\left(e^{\frac{\omega}{2T_R}} - 1\right)}\,, \tag{4.8}$$

where A_h is the horizon area, the left- and right-moving temperatures were defined in (2.45) and

$$\frac{1}{T_H} = \frac{1}{2}\left(\frac{1}{T_L} + \frac{1}{T_R}\right), \tag{4.9}$$

is the inverse of Hawking temperature.

4.2. D-brane calculation

Now we calculate the absorption probability for incoming scalar particles when our D-brane system is in the vacuum state. The canonically normalised fields are $\tilde{\Phi} = \Phi/k_6$

[5] If $\Delta\theta \sim 1$ which happens for the D5-D1 brane system (with $f = 16$) the near and far regions do not overlap. In this case there are ωr_i corrections which are suppressed within our approximation [6].

and $\tilde{\zeta}^r = \sqrt{T_{ins}}\zeta^r$. These fields have the following mode expansion:

$$
\begin{aligned}
\tilde{\zeta}^r &= \sum_q \frac{1}{\sqrt{2L_{eff}q_0}}\left(\zeta_q^r e^{iq_\sigma x^\sigma} + \zeta_q^{r\dagger} e^{-iq_\sigma x^\sigma}\right), \\
\tilde{\Phi} &= \sum_{k_1,\vec{k}} \frac{1}{\sqrt{2L_1\mathcal{V}_4 k_0}}\left(\Phi_k e^{ik\cdot x} + \Phi_k^\dagger e^{-ik\cdot x}\right),
\end{aligned}
\tag{4.10}
$$

where q and k_1 are the corresponding momenta along the string direction and $\vec{k}$ the momentum in the transverse space with volume $\mathcal{V}_4$. Note that we are considering a six-dimensional free action for the field Φ that arises from compactification of the IIB theory on T^4 and not the five-dimensional action (3.2). The dependence on the string direction will in fact be irrelevant because we shall consider modes of the field satisfying $k_1 = 0$, i.e. we do not consider charged particles [6, 8]. However, it is important to note that the scalar particle is defined on a length $L_1 = 2\pi R_1$ while the instanton strings on a length $L_{eff} = L_1 N_1 N_2$. We have normalised the states such that $|q\rangle = \zeta_q^{r\dagger}|0\rangle$ represents a single particle with momentum q in the length L_{eff} and $|k\rangle = \Phi_k^\dagger|0\rangle$ a single particle with momentum k in the volume $L_1\mathcal{V}_4$. Thus, from the spacetime perspective (i.e. integrating over the string direction) a state $|k\rangle$ carries a flux $1/\mathcal{V}_4$.

In terms of the canonically normalised fields the interacting vertex (3.11) becomes

$$
S_{int} = \frac{k_6}{2}\int dt \int_0^{L_{eff}} dx^1 \tilde{\Phi} \sum_{r=1}^{4n_L\bar{n}_L} \partial_\sigma\tilde{\zeta}^r\partial^\sigma\tilde{\zeta}^r. \tag{4.11}
$$

The initial and final states for the process considered are

$$
\begin{aligned}
|i\rangle &= \Phi_k^\dagger|0\rangle, & k &= (k_0, 0, \vec{k}), \\
|f\rangle &= \zeta_q^{r\dagger}\zeta_p^{r\dagger}|0\rangle, & q &= (q_0, q_1), \quad p = (p_0, p_1).
\end{aligned}
\tag{4.12}
$$

The amplitude for this process is then

$$
T_{fi} = -\frac{k_6}{2}\frac{p\cdot q}{\sqrt{2L_1\mathcal{V}_4 k_0}\sqrt{2L_{eff}q_0}\sqrt{2L_{eff}p_0}}2. \tag{4.13}
$$

The reason the supersymmetric completion of (4.11) was not considered is that on-shell fermions give a vanishing contribution to this amplitude. The final factor of two is because either of the ζ's in (4.11) may annihilate either of the final particle states in (4.12) [5]. The probability per unit of time for this transition to occur is then

$$
\Gamma_{fi} = L_{eff}(2\pi)^2\delta(k_0 - p_0 - q_0)\delta(p_1 + q_1)|T_{fi}|^2. \tag{4.14}
$$

To obtain the total probability rate we have to sum over the $4n_L\bar{n}_L$ species of particles and integrate over the final momenta dividing by two due to particle identity. The result is

$$
\Gamma_{abs} = 4n_L\bar{n}_L\frac{1}{2}\sum_{p,q}\Gamma_{fi} = \frac{1}{4\mathcal{V}_4}L_{eff}n_L\bar{n}_L k_5^2\omega\,. \tag{4.15}
$$

JHEP11(1998)007

Since the state $|i\rangle = |k\rangle$ carries the flux $1/\mathcal{V}_4$ we have that $\sigma_{abs} = \mathcal{V}_4\Gamma_{abs}$ which agrees exactly with (4.7). Besides the rather successfully D-3-brane case [19, 20, 21] this is the first example where this calculation has been done by deducing from first principles the coupling between the bulk field and the worldvolume fields.

The calculation when the D-brane system is described by a thermal state of left- and right-movers is done in the following way. We consider a unit normalised state $|n_R, n_L\rangle$ of the instanton strings with $n(p_R)$ and $n(p_L)$ right- and left-mover occupation numbers. Now the initial and final states for the process are

$$\begin{aligned} |i\rangle &= \Phi_k^\dagger |n_R, n_L\rangle\,, & k &= (k_0, 0, \vec{k})\,, \\ |f\rangle &= \zeta_q^{r\dagger}\zeta_p^{r\dagger}|n_R, n_L\rangle\,, & q &= (q_0, q_1)\,, \quad p = (p_0, p_1)\,. \end{aligned} \tag{4.16}$$

The amplitude for this process is

$$T_{fi} = -\frac{k_6}{2}\frac{p\cdot q}{\sqrt{2L_1\mathcal{V}_4 k_0}\sqrt{2L_{eff}q_0}\sqrt{2L_{eff}p_0}} 2(n(p)+1)(n(q)+1)\,. \tag{4.17}$$

The total probability rate is obtained by summing over all final states and averaging over all initial states in the thermal ensemble [7]. This gives the desirable Bose-Einstein thermal factors. Agreement with (4.8) is found by using $\sigma_{abs} = V_4(\Gamma_{abs} - \Gamma_{emis})$, where Γ_{emis} is the probability rate for the time reversed process [9].

5. CFT/AdS duality

In this section we start by analysing the region of validity of the previous cross section calculations. We shall define a double scaling limit [19] where the supergravity cross section calculation and our gauge theory calculation of the D-brane absorption probability should in fact agree. The last subsections are concerned with the near horizon geometry and Maldacena's duality proposal [25].

5.1. Double scaling limit

Consider the ground state of our D-brane system. We argued in subsection 2.2 that the supergravity approximation holds if the length scales in the solution are big in string units. The resulting condition is

$$r_i^2 \gg r_1 r_2 \sin\Delta\theta \gg 1 \quad\Rightarrow\quad g_{eff}\sin\Delta\theta \gg 1\,, \tag{5.1}$$

where g_{eff} is the D-brane effective string coupling defined in (2.6). In this limit string corrections to the metric are suppressed (the string loop corrections are also suppressed because we are considering $g \ll 1$). The curvature of this background is bounded by its value at $r = 0$ where

$$\text{curv} \sim \frac{1}{r_1 r_2 \sin\Delta\theta} \sim \frac{1}{g_{eff}\alpha' \sin\Delta\theta}\,. \tag{5.2}$$

The classical cross section is naturally expanded in powers of $\omega^4 \text{curv}^{-2}$. Thus, for energies such that

$$\omega^4 (g_{eff} \alpha' \sin \Delta\theta)^2 \ll 1 \,, \tag{5.3}$$

we expect the classical approximation to the scattering process to be good. Both conditions (5.1) and (5.3) are satisfied in the double scaling limit [19]

$$g_{eff} \sin \Delta\theta \to \infty \,, \qquad \omega^4 \alpha'^2 \to 0 \,, \tag{5.4}$$

such that

$$(g_{eff} \sin \Delta\theta)^2 \left(\omega^4 \alpha'^2\right) , \tag{5.5}$$

is held fixed and small, i.e. (5.3) holds. The second condition in (5.4) implies that the massive excitations on the D-5-brane may be neglected when comparing with the supergravity cross section calculation, i.e. it corresponds to the decoupling limit of the brane theory. These states have a mass that scales as $1/\sqrt{\alpha'} \gg \omega$ (note that some of the massive states have a mass proportional to $\sqrt{\Delta\theta}$ which is small but it is held fixed in the limit (5.4)). This is the reason why they were dropped in the field theory description. They may be neglected in the double scaling limit.

Now we show that in the limit (5.4) the D-brane calculation may in fact be trusted (even if $g_{eff} \to \infty$). The only scale in the scattering calculation is given by the gravitational coupling k_6 as may be seen in the interacting Lagrangian when written in terms of the canonically normalised fields. The cross section is then an expansion in powers of

$$\omega^4 k_6^2 n_L \bar{n}_L \frac{L_{eff}}{L_1} \,. \tag{5.6}$$

The $n_L \bar{n}_L$ factor is because we sum over all different species in the final state and the $L_{eff}/L_1 = N_1 N_2$ factor because the scalar particles leave in a length L_1 while the instanton strings in a length L_{eff} (the state $|k\rangle = \Phi_k^\dagger |0\rangle$ corresponds to a single particle in the volume $L_1 \mathcal{V}_4$ or L_{eff}/L_1 particles in the volume $L_{eff} \mathcal{V}_4$). The ω^4 factor follows from dimensional analysis. Thus, the perturbative string calculation is valid for

$$\omega^4 N_1 N_2 n_L \bar{n}_L k_6^2 \ll 1 \quad \Rightarrow \quad \omega^4 \left(\alpha' g_{eff} \sin \Delta\theta\right)^2 \ll 1 \,, \tag{5.7}$$

which holds in the limit (5.4).

We conclude that both the classical and string cross section calculations have an overlapping domain of validity and it is therefore not surprising that agreement is found.

5.2. Near horizon geometry

As it is the case for the D5-D1 brane configuration [54, 55, 56] the near horizon geometry associated with our D-brane bound state is $AdS_3 \times S^3 \times M$, where M is a compact manifold (T^4 in our case). Taking the limit $r \to 0$ we obtained the following fields describing the near horizon geometry

$$
\begin{aligned}
ds_{10}^2 &\sim ds_3^2 + \frac{R^2}{r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2} ds^2(T^4) + R^2 d\Omega_3^2\,, \\
ds_3^2 &= -\frac{\rho^2}{R^2}d\tau^2 + \frac{R^2}{\rho^2}d\rho^2 + \rho^2 d\varphi^2, \\
e^{2\phi} &\sim \frac{R^4}{(r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2)^2}\,, \\
\mathcal{F}_3 &\sim 2(r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2)\epsilon_3\,,
\end{aligned}
\tag{5.8}
$$

where $R^2 = r_1 r_2 \sin\Delta\theta$, $\tau = \frac{R}{R_1}t$, $\varphi = \frac{x_1}{R_1}$, $\rho = \frac{R_1}{R}r$ and ϵ_3 is the unit 3-sphere volume form. To obtain the horizon value for $\mathcal{F}_3$ we used the behaviour at the horizon of the Chern-Simons terms in the solution. Note that the electric terms in $\mathcal{F}_3$ as well as $\mathcal{H}$ and $\mathcal{F}_5$ vanish at the horizon. R is the 3-sphere radius and the AdS_3 cosmological constant is given by $\Lambda = -R^{-2}$. This geometry is interpreted as the R-R ground state of string theory on the $AdS_3 \times S^3 \times T^4$ background [57, 58].

For later convenience we express the parameters in the solution (5.8) in terms of the field theory quantities

$$
\begin{aligned}
R^2 &= g\alpha'\sqrt{N_1N_2n_L\bar{n}_L}\sqrt{\frac{\alpha'^2}{R_2\ldots R_5}} \sim g\alpha'\sqrt{N_1N_2n_L\bar{n}_L} \equiv g\alpha'\sqrt{Q_1Q_5}\,, \\
\mathcal{F}_3 &\sim 2g\alpha'(N_1p_1\bar{p}_1 + N_2p_2\bar{p}_2)\epsilon_3 \equiv 2g\alpha' Q_5\epsilon_3\,, \\
v_f &\equiv \frac{V_f(T^4)}{(2\pi)^4\alpha'^2} = \frac{N_1N_2n_L\bar{n}_L}{(N_1p_1\bar{p}_1 + N_2p_2\bar{p}_2)^2} \equiv \frac{Q_1}{Q_5}\,.
\end{aligned}
\tag{5.9}
$$

The last identifications in these equations correspond to the D5-D1 brane case. The T^4 volume has its fixed value at the horizon while the six-dimensional string coupling g_6 has the same value as in the original solution where it was constant [25].

For the non-extreme solution the 3-dimensional geometry in (5.8) valid in the near horizon region is replaced by the BTZ black hole [59]. In the previous (τ, φ) coordinates and defining a new radial coordinate $\rho^2 = R_1^2(r^2 + r_0^2\sinh^2\beta)/R^2$ [63] the resulting metric reads

$$
\begin{aligned}
ds_3^2 &= -N^2 d\tau^2 + N^{-2}d\rho^2 + \rho^2\left(d\varphi - N_\varphi d\tau\right)^2, \\
N^2 &= \frac{\rho^2}{R^2} - \frac{R_1^2 r_0^2\cosh 2\beta}{R^4} + \frac{R_1^4 r_0^4\sinh 2\beta}{4R^6\rho^2}\,, \qquad N_\varphi = \frac{R_1^2 r_0^2\sinh 2\beta}{2R^3\rho^2}\,.
\end{aligned}
\tag{5.10}
$$

The derivation of this metric assumes that we are in the dilute gas regime and that the condition to neglect the massive string states is satisfied, i.e.

$$
r^2,\ r_n^2,\ r_0^2 \ll r_i^2\tan^2\theta_i\,,\ r_1r_2\sin\Delta\theta\,,
\tag{5.11}
$$

where we are assuming that we are close enough to the horizon such that r satisfies these conditions. This geometry corresponds to an excited (thermal state) of string theory on the $AdS_3 \times S^3 \times T^4$ background [25]. Using the formulation of quantum gravity in $2+1$ dimensions as a topological Chern-Simons theory [60, 61], Carlip found that the degrees of freedom at the horizon are described by a $1+1$ conformal field theory reproducing the entropy formula for the BTZ black hole [62]. A different approach originally due to Strominger [58, 63] is based on the fact that any quantum theory of gravity on AdS_3 has an asymptotic algebra of diffeomorphisms given by the Virasoro algebra [57].[6] Physical states will form representations of such algebra and the correct entropy formula follows (for the correct central charge). Of course all these results are valid in our model because the near horizon geometry is similar to the D5-D1 brane case. The only difference is the way we parametrise the solutions.

5.3. CFT/AdS correspondence

The motivation for the conjectured duality between the decoupled theory on the brane and supergravity on the $AdS_3 \times S^3 \times T^4$ background [25] relies in part on the agreement between the entropy calculations and the scattering calculations in the double scaling limit (5.4). The region of validity of the supergravity phase is given in eq. (5.1) which reads

$$\frac{R^2}{\alpha'} \sim g\sqrt{N_1 N_2 n_L \bar{n}_L} \gg 1\,. \tag{5.12}$$

This may be accomplished by taking the $\alpha' \to 0$ limit

$$g_{YM}^2 = (2\pi)^3 g\alpha' \to 0\,, \qquad N_1 \sim N_2 \to \infty\,, \tag{5.13}$$

such that

$$R^2 \sim g_{YM}^2\sqrt{N_1 N_2 n_L \bar{n}_L}\,, \tag{5.14}$$

is held fixed. Note that in this limit all the fields in (5.8) are held fixed as may be seen from (5.9). This limit is equivalent to the double scaling limit (5.4), the difference is that the energies are held fixed and $\alpha' \to 0$.

Now in the D-brane side the limit (5.13) is just the 't Hooft large N limit. The advantage of formulating Maldacena's duality conjecture using this model is that we know, at least in principle, the action for the D-5-brane and its coupling to the bulk fields. As in the analysis given by Alwis for the D-3-brane [81] we consider the 't Hooft scaling for the D-5-brane action and see what conclusions we may draw. Schematically 't Hooft scaling may be analysed by writing (the factor $\sqrt{N_1 N_2}$ replaces the usual factor of N because we are considering the Higgs branch of the theory and the fields b_α are in the fundamental representation of $U(N_1 p_1 \bar{p}_1) \otimes U(\overline{N_2 p_2 \bar{p}_2})$)

$$S \sim -\frac{\sqrt{N_1 N_2}}{\alpha'^2}\frac{\alpha'^2}{\sqrt{N_1 N_2}\, g_{YM}^2}\int d^6x \left[\operatorname{tr} G^2 + (2\pi\alpha')^2 \operatorname{tr} G^4 + \cdots\right]. \tag{5.15}$$

[6] See refs. [64]–[80] for recent work on the subject.

Rescaling the fields as

$$G \sim \frac{(N_1 N_2)^{1/4} g_{YM}}{\alpha'} \tilde{G} \sim \frac{R}{\alpha'} \tilde{G} \,, \qquad \tilde{G} = d\tilde{B} + \frac{R}{\alpha'} [\tilde{B}, \tilde{B}] \,, \tag{5.16}$$

we obtain the action

$$S \sim -\sqrt{N_1 N_2} \frac{1}{\alpha'^2} \int d^6 x \left[\operatorname{tr} \tilde{G}^2 + R^2 \operatorname{tr} \tilde{G}^4 + \cdots \right]. \tag{5.17}$$

Note that the background field $\tilde{G}^0$ remains finite, i.e. $\tilde{G}^0 \sim \mathrm{diag}(\tan\theta_1, \ldots)/R$. The $1/\alpha'^2$ factor in the front of the action is important because we are compactifying the theory on T^4 with a volume $V_4 \sim \alpha'^2$ and therefore it ensures that the action remains well defined in the limit $\alpha' \to 0$. We are keeping $|\tan\theta_i| \ll 1$ such that our gauge theory fluctuating spectra do not suffer from DBI and derivative corrections. However, for processes involving energies such that $E \sim 1/R$ there will be DBI corrections [81]. In the infrared limit $E \ll 1/R$ we recover the SYM description and after reduction to $1+1$ dimensions we recover the superconformal limit in the original derivation of the duality [25]. Also, this limit corresponds in the supergravity side to the $r \to 0$ limit and we recover the near horizon geometry (moving in r corresponds to moving in the energy scales in the field theory side [25]).

Our model gives a definite proposal for the conformal theory and for the coupling of the conformal fields to the bulk fields on the AdS_3 boundary. In other words Maldacena's duality proposal may be recasted in the following form: *The Higgs branch of the large N limit of six-dimensional SYM theory compactified on T^4 with a 't Hooft twist is dual to supergravity on $AdS_3 \times S^3 \times T^4$.* The parameters relating the dual theories have already been explained. The coupling to the bulk fields is determined by the DBI action and by requiring conformal invariance.

A more precise formulation of the duality conjecture was given by means of calculating conformal field theory correlators using the supergravity near horizon geometry [26, 27]. Unfortunately the number of calculations that may be done to test this conjecture is very limited because in the overlapping domain of validity of the dual theories the 't Hooft coupling of the gauge theory is very large. Also, one would like to investigate whether this duality is carried away from the conformal and near horizon limits. In this case we need the full strongly coupled DBI action. Our model is a starting point to perform such computations in parallel with the D-3-brane case [82, 83] (see also [84, 85] for work on the D5-D1 brane system).

In the following we shall argue that in the 't Hooft limit the tension of the instanton strings in (2.30) gets normalised and it scales as

$$T_{eff} \sim \frac{1}{R^2} \sim \frac{1}{2\pi\alpha' g \sqrt{N_1 N_2 n_L \bar{n}_L}} \equiv \frac{1}{2\pi\alpha' g \sqrt{Q_5 Q_1}} \,, \tag{5.18}$$

confirming the results in [15, 16, 28]. The argument is rather heuristic and by no means rigourous. The compactification of the action (5.17) gives a bosonic action of the type

$$S = -\sqrt{N_1 N_2} \int d^2x \left(G_{rs}(\tilde{\zeta}) \partial_\sigma \tilde{\zeta}^r \partial^\sigma \tilde{\zeta}^s + \cdots \right), \tag{5.19}$$

where the dots denote the DBI corrections and the fields $\tilde{\zeta}$ are dimensionless. Now it is hoped that in the limit $N_i \to \infty$ the Feynman rules that follow from this action should define an effective action reproducing such rules. Of course this will involve very large Feynman graphs because the 't Hooft coupling is becoming very large. Such effective action should be identified with the rather successfully effective string action used in the computations of scattering amplitudes. In this limit the only scale in the problem is R. We are forced to conclude that the effective string tension scales as in (5.18). Note that we are arguing here that it is the gauge field A_α that is associated with the effective string description. An opposite point of view was advocated in [81]. One could argue that a tension like

$$T_{eff} \sim \frac{1}{2\pi\alpha' g(N_1 p_1 \bar{p}_1 + N_2 p_2 \bar{p}_2)} \sim \frac{1}{2\pi\alpha' g Q_5}, \tag{5.20}$$

would remain finite in the large N limit. This is in fact true. However it is difficult to see how it would arise when considering the Feynman rules that should originate the effective string action defined above. The reasons are: Firstly, the fields $b_{\hat{\alpha}}$ associated with the instanton strings transform under the fundamental representation of $U(N_1 p_1 \bar{p}_1) \otimes U(\overline{N_2 p_2 \bar{p}_2})$, therefore the trace of any gauge invariant combination of these fields depends on N_i through a power of $N_1 N_2$ only; Secondly, all couplings in the action (5.17) depend on N_i in the same way. It follows that any scattering amplitude is bound to depend on N_i through the particular combination $N_1 N_2$.

6. Conclusion

Let us start by summarising our results. We have argued that a model based on D-5-branes with a constant self-dual field strength on T^4 describes 5-dimensional black holes within a perturbative string theory framework and that the D5-D1 system constitutes a special case of this model. The fluctuating spectra of this bound state is described by Polchinski's open strings ending on the D-5-branes. This means that the Higgs branch of the theory, which describes the 'internal' excitations of the bound state, is associated with fundamental modes of the worldvolume fields. We may take the volume of the T^4 to satisfy $V_4 \sim \alpha'$ while string derivative corrections are negligible. The explicite knowledge of the microscopics of the D-brane bound state allowed us to make a definite proposal for the conformal field theory governing the Higgs branch of the theory. This means that we may deduce from first principles the coupling of the bulk fields to the worldvolume fields. We have done this for a minimally coupled scalar and find agreement with the supergravity scattering cross section calculation. Also, the explicite knowledge of this conformal theory is relevant for Maldacena's duality proposal. We think that our model could be a starting point to the investigation of the field theory side of this duality conjecture.

Regretfully we have left for the future a number of calculations that should prove or disprove the validity of our approach to black hole dynamics. One should calculate the coupling of the instanton strings to the minimally coupled scalars arising from the internal metric on the T^4 as well as to the fixed scalars. In these cases we expect that the fermions will contribute to the corresponding scattering processes. We may use fermionisation of the bosonic action or find the supertorons, i.e. the fermionic partners of the toronic excitations associated with the fields b_α (and d_m). An alternative approach to this problem is to use the string description of the D-brane bound state and to calculate the corresponding scattering amplitude using string techniques. This will involve the usual disk diagram with three vertices (two of which are on the disk boundary). One should also generalise the D-5-brane bound state so that it allows $f = 4$ in the D5-D1 brane bound state case. It would also be interesting to reproduce this bound state while suppressing the DBI corrections. Another interesting problem is to find the hyperkähler metric G_{rs} determining the superconformal field theory of the instanton strings . This would provide us with a better understanding of the interacting theory. Another problem is to consider a D-5-brane configuration with instantons and anti-instanton [38] (in this case there are tachyonic modes in the spectra which signal the instability of the configuration). Knowledge of the interacting theory could be relevant to understand the entropy formula away from extremality and the dilute gas regime. One would also like to generalise the field theory description with a 't Hooft twist to other compact manifolds, for example $K3$.

Hopefully, a better understanding of this model will shed light to the string theory approach to black hole physics.

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Note added

After this work appeared as a preprint I learned that a similar calculation to the one presented in sections 3 and 4 for the D-brane emission rates using the DBI action was carried out by S.D. Mathur [86].

JHEP11(1998)007

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